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Robust and Efficient Trimming Algorithm for Application to Advanced Mathematical Models of Rotorcraft

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Nomenclature

c	=	control state vector
J	=	Jacobian matrix
n	=	number of blades per rotor
P_v, P_r	=	vehicle overall permutation matrix, rotor permutation matrix
P_6	=	permutation matrix for a three-bladed rotor with two flapping states per blade
p, q, r	=	fuselage roll, pitch, and yaw rates about body axis set
$p_{ifr(l)}$, $q_{ifr(l)}$	=	harmonic induced flow components for right (left) rotor
s	=	vehicle state vector
s_{fs}, s_{if}, $s_{rr(l)}$	=	body axis flight state, induced flow, and right (left) rotor state vectors
t_p	=	time for one period of oscillation in rotor forces and moments
u_a, v_a, w_a	=	fuselage velocity components along x, y, and z body axis, respectively
V	=	magnitude of vehicle speed
$w_{ifr(l)}$	=	uniform induced flow component for right (left) rotor
x, $\bar{x}$, x_{trim}	=	flight state vector, mean flight state vector, and specified mean flight state vector
β	=	blade flapping angle, fuselage sideslip angle
γ, θ, ϕ	=	fuselage angles of climb, pitch, and roll
θ_{0c}, θ_{0d}	=	combined and differential collective inputs
θ_{1cc}	=	combined lateral cyclic
θ_{1sc}, θ_{1sd}	=	combined and differential longitudinal cyclic inputs
Ψ_r	=	amount of rotor revolution from initial position
Ω	=	fuselage turn rate

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I. Introduction

THE latest generation of rotorcraft simulations offers enhanced fidelity by considering the behavior of each blade individually when evaluating the rotor forces and moments.[1–3] When using such an approach, the trim condition achieved with a constant control position is no longer time-invariant, but has an oscillatory component. Therefore, when seeking a trim condition it is necessary to look for a periodic solution to the equations of motion which, in the mean, satisfies the stipulated flight conditions.

All currently published methods require at least one full revolution of the rotor to investigate the periodicity of each state, and are based on a two-stage approach where an outer iteration adjusts the control positions to achieve the specified mean flight condition.[4–6] Damping parameters for the iteration are utilized by some existing methods since divergence remains a problem when trimming to adverse areas of the flight envelope.

The algorithm described in this Note achieves economy and reliability in two ways. Firstly, it uses a single stage of iteration to establish simultaneously both periodicity and the specified mean flight condition. Secondly, it exploits the fact that in trim each rotor blade follows exactly the same trajectory relative to the vehicle. Thus, for an n-bladed rotor, the whole trajectory is established by considering only an nth part of a complete revolution. The incorporation of these principles into a modified periodic shooting algorithm leads to an efficient and reliable method of achieving trim.

The method has been applied to an individual blade simulation (GTILT), which has been developed at the University of Glasgow to model a generic tilt-rotor configuration.[1] Tilt-rotor models present significant difficulties from the trimming perspective due to the vehicle's control redundancy and ability to fly in helicopter, transitional, and aeroplane modes. Despite this, rapid convergence to trim has been achieved with the method, even in adverse areas of the flight envelope. There has been no recourse to the use of damping in the iteration scheme since convergence has not been sensitive to its initial values. For a conventional helicopter, the algorithm is directly applicable to a configuration where the rates of rotation of main and tail rotors are related by a simple ratio; it is also applicable when a rotor disc representation for the tail rotor is used—as is commonly the case.

II. Definition of Periodic Trim

The period of oscillation in the trimmed rotor forces and moments is dependent upon the number of blades in the rotor. Each azimuthal position has its own associated blade pitch and aerodynamic velocity, so that, for identical blades, each blade will generate the same contribution to the rotor forces and moments as it passes through that position. Thus, an n-bladed rotor has to rotate through $2\pi/n$ rad to have had, instantaneously, a blade in all azimuthal positions. Therefore, the full period of trimmed rotor forces and moments is described in $2\pi/n$ rad of revolution. The same period will apply to the vehicle's body axis flight states, thus, when trim has been achieved, Eq. (1) is satisfied

$$s_{fs}(0) = s_{fs}(2\pi/n) \tag{1}$$

where the vehicle flight state vector s_{fs} is measured in body axis and is given by

$$s_{fs} = [u_a \quad v_a \quad w_a \quad p \quad q \quad r \quad \theta \quad \phi]^T$$

The dynamic inflow model included in GTILT is that of Peters and HaQuang.[7] It employs a nonlinear, three-state, first-order differential equation to predict the rotor induced flow. Since the induced flow is derived from the rotor thrust and pitching and rolling moments, the periodicity of the induced flow states is also $2\pi/n$, as shown in Eq. (2)

$$s_{if}(0) = s_{if}(2\pi/n) \tag{2}$$

where the state vector of induced flow s_{if} for a vehicle with two rotors is given by

$$s_{if} = [w_{ifr} \quad p_{ifr} \quad q_{ifr} \quad w_{ifl} \quad p_{ifl} \quad q_{ifl}]^T$$

For each individual blade to have passed through all azimuthal locations the rotor must have completed a full revolution, so that, in trim, the periodicity of an individual blade is expressed in Eq. (3)

$$s_r(0) = s_r(2\pi) \tag{3}$$

where the state vector s_r, for a rotor with two flapping states per blade is given by

$$s_r = [\beta_1 \quad \dot{\beta}_1 \quad \beta_2 \quad \dot{\beta}_2 \quad : \quad : \quad \beta_n \quad \dot{\beta}_n]^T$$

From Eqs. (1–3) it is evident that a rotation through $2\pi/n$ rad is sufficient to examine periodicity of the fuselage and induced flow states, whereas a full rotation, 2π rad, is necessary to confirm the periodicity of an individual blade. If one considers the full set of blades, then the definition of rotor trim given by Eq. (3) can be reformulated and expressed as a requirement over a rotation through $2\pi/n$ rad. In trim, the blades must trace out exactly the same trajectory as they advance around the azimuth; the full period of this trajectory being described in 2π rad of revolution. However, a phase shift of $2\pi/n$ rad occurs between the paths followed by adjacent blades, consequently, one need only integrate the rotor through $2\pi/n$ rad to have a full description of the complete trajectory traced out by a trimmed blade. Furthermore, for a rotor in trim, the states of an arbitrary blade m, at $\Psi_r = 2\pi/n$ rad will map onto the initial states of an identical blade $m + 1$ when $\Psi_r = 0$. This feature is exploited to form a definition of rotor trim using only $2\pi/n$ rad of rotor revolution with a permutation matrix representing this mapping

$$s_r(2\pi/n) = P_r s_r(0) \tag{4}$$

where the general form of P_r is given by

$$P_r = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & . & . & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & . & . & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & . & . & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & . & . & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & . & . & 0 & 0 \end{bmatrix}$$

It is clear that the permutation matrix in this case is the identity matrix with the nonzero elements shifted to the right by an amount corresponding to the number of states per blade. More states per blade could easily be accommodated by the inclusion of an appropriate permutation matrix.

The expressions given in Eqs. (1), (2), and (4) can be combined to define the overall vehicle trim. For a vehicle with two three-bladed rotors, three inflow states per rotor, and two flapping states per blade, the overall definition of the periodicity of vehicle trim can be expressed in terms of the relationship between the state vector of the whole vehicle at the start and end of a rotation through $2\pi/n$ rad

$$s(2\pi/n) = P_v s(0) \tag{5}$$

where P_v is given by

$$P_v = \begin{bmatrix} P_6 & 0 & 0 & 0 \\ 0 & P_6 & 0 & 0 \\ 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & I_8 \end{bmatrix}$$

for the tilt-rotor, the vehicle state vector is given by

$$s = \begin{bmatrix} s_{rr} \\ s_{rl} \\ s_{if} \\ s_{fs} \end{bmatrix}$$

and P_6 is a 6×6 permutation matrix of the form given in Eq. (4), I_m is the $m \times m$ identity matrix, and s_{rr} and s_{rl} are the state vectors for the right and left rotors, respectively.

III. Specification and Convergence of Periodic Tilt-Rotor Trim

The required mean flight condition is specified through the total velocity, fuselage sideslip angle, fuselage angle of climb, turn rate, and fuselage bank (or roll) angle. (The bank angle is available because of the additional DOF accorded by the tilt rotor configuration in helicopter mode.) At each iteration, the time-averaged integral of the current flight state vector is evaluated to yield the current mean

$$\bar{x} = \frac{1}{t_p}\int_0^{t_p} x \, \mathrm{d}t$$

where

$$x = [V \quad \beta \quad \gamma \quad \Omega \quad \phi]^T$$

and, on convergence, the current mean equals the specified state vector

$$\bar{x} = x_{\mathrm{trim}} \tag{6}$$

Together, Eqs. (5) and (6) define the conditions that must be satisfied to achieve a specified trim state. They constitute 31 nonlinear equations for the 26 initial values $s(0)$, and 5 control positions c. A linearization derived from a Taylor expansion about the solution values results in the form of Newton-Raphson iteration specific to this application, which is shown in Eq. (7):

$$\begin{bmatrix} s(0) \\ c \end{bmatrix}_{i+1} = \begin{bmatrix} s(0) \\ c \end{bmatrix}_i - \begin{bmatrix} J_{11} - P_v & J_{12} \\ J_{21} & J_{22} \end{bmatrix}_i^{-1} \times \begin{bmatrix} s(2\pi/n) - P_v s(0) \\ \bar{x} - x_{\mathrm{trim}} \end{bmatrix}_i \tag{7}$$

where, for the tilt-rotor, the control state vector is given by

$$c = [\theta_{0c} \quad \theta_{od} \quad \theta_{lsc} \quad \theta_{lsd} \quad \theta_{lcc}]^T$$

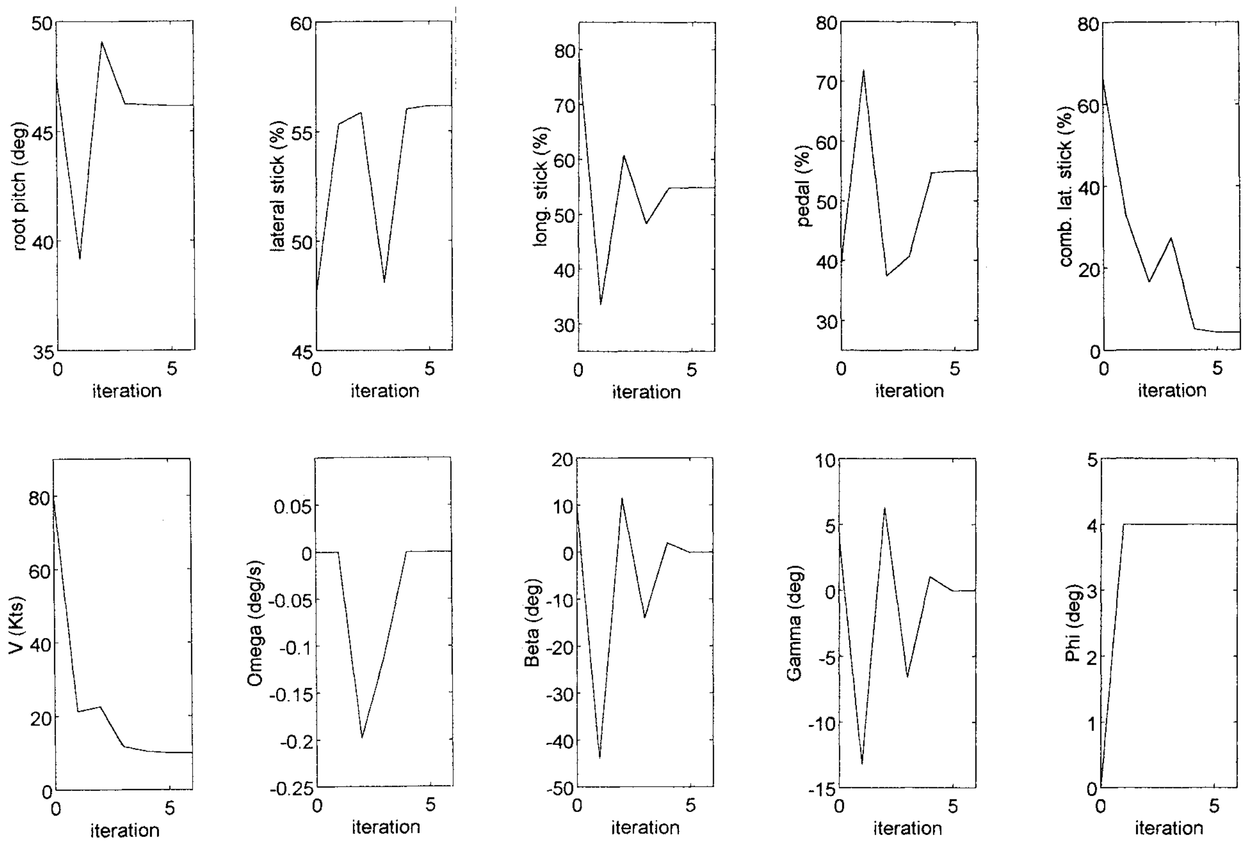

Fig. 1 Flight path and control state iteration histories for 10-kt trimmed flight with 4 deg of bank in helicopter mode.

The iteration is a single-stage method that simultaneously solves the equations to provide both periodicity and the specified mean flight condition. No modification of the basic iteration, through the introduction of damping or acceleration parameters, for example, has been found to be necessary. Its performance is considered in the next section.

IV. Performance of Periodic Trimmer

Control and flight-path iteration histories produced during a trim to 10-kt forward flight with 4-deg bank angle in helicopter mode are given in Fig. 1 (the iteration histories produced concurrently for the initial conditions are not shown). As can be seen, rapid convergence is obtained with six iterations producing the specified trim, despite the fact that a relatively poor set of initial values was used. Displacements of all five controls are necessary to produce this trim with the converged blend of controls being qualitatively valid and described in Ref. 8 as the lateral translation mode.

Figure 1 typifies the convergence produced by this trimming method, i.e., a maximum of six iterations is usually sufficient to produce a specified trim state, even in adverse areas of the envelope. From Fig. 1 it is evident that the specified trim has been produced despite the fact that a relatively poor set of initial values was used. In fact, this scheme is particularly robust to the quality of start values and, consequently, there has been no requirement to include damping parameters.

When using this methodology to trim a simulation with three blades per rotor, the rotor must be integrated through $2\pi/3$ rad to form each column of the Jacobian matrix. Consequently, for a model with 31 states, a typical trim of four iterations would require $41\frac{1}{3}$ revolutions of rotor azimuth to complete; for the XV-15 in helicopter mode this corresponds to approximately 4.3 s of real-time simulation. This technique produces convergence considerably faster (by a factor equal to the number of blades in the rotor) than equivalent methods that utilize one full revolution of the rotor to form each column of the Jacobian matrix. The improvement is further enhanced by the fact that there has been no recourse to a two-phase approach, nor the inclusion of damping parameters that would adversely affect the performance of the scheme.

Those iterative techniques that disregard the periodic property of the trim state and rely on integrating the equations of motion until the transients sufficiently decay have their performance dictated by the rate of decay of the most persistent natural mode. Since rotorcraft typically possess at least one lightly damped mode, such methods are usually several orders of magnitude slower to produce convergence than those that exploit the inherent periodicity of the equations.

V. Conclusions

This Note presents an algorithm that is capable of reliably trimming advanced rotorcraft simulations to a wide range of periodic trim states. The algorithm provides enhanced performance over existing methodologies by exploiting the following features:

1) A single iterative loop solves simultaneously for the initial conditions and control displacements necessary to produce a specified periodic flight state.

2) Only a partial revolution of rotor azimuth is used to form each column of the Jacobian matrix.

3) Acceptable robustness of convergence is achievable without the use of damping parameters.

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Performance Benefit of Second Harmonic Control in Helicopters

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I. Introduction

THE classical method to control a helicopter rotor, which is still predominant, entails zero and first harmonic inputs introduced with a swashplate. It has long been recognized[1–3] that second and higher harmonics hold a potential for performance improvement. The idea is to unload the retreating sector of the disk so as to avoid blade stall. The advancing sector must also be unloaded for balance, in the process alleviating the drag rise due to compressibility. Increased load is then carried by the fore and aft sectors.

Fly-by-wire and fly-by-light control, being exploited in contemporary design of fixed wing aircraft, can provide for higher harmonic control (as well as individual blade control) in helicopters. A number of recent patents cover such applications.[4–10]

The analysis of rotor dynamics, the effect of higher harmonic control included, used to be cumbersome and to require far-reaching simplifications. This is no longer the case. Recent advances in microprocessor technology make it possible to simulate rotor blade motion accurately. This Note exploits a blade element model[11,12] to explore the potential performance benefit of second harmonic control. The specific effect sought is relief of retreating blade stall in forward flight. We demonstrate by computer simulation that second harmonic control reduces power consumption and makes flight possible at speeds that up to now could not be sustained at any power level.

For comparison and continuity we treat the same sample helicopter that Arcidiacono addressed.[3] Our results roughly bear out his. However, unlike the previous treatment, it is no longer assumed that the relationship between control input and flapping response is linear. We determine the optimal second harmonic control input as a function of forward speed and arrive at a top speed more modest than Ref. 3.

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II. Higher Harmonic Rotor Control

In a stationary rotor state (steady flight condition), a periodic blade pitch is required. We consider a commanded pitch θ_c that is a function of the azimuth angle ψ. We then consider variation of this commanded pitch law for control of the helicopter.

The commanded pitch may be expanded in a Fourier series

$$\theta_c(\psi) = \sum_{n=0}^{\infty} [A_n \cos(n\psi) + B_n \sin(n\psi)] \tag{1}$$

With a conventional swashplate arrangement, A_0, A_1, B_1 are the pilot's collective pitch, lateral cyclic pitch, and longitudinal cyclic pitch inputs, respectively. All terms with $n > 1$ vanish. If higher-order terms were introduced, A_0, A_1, and B_1 would still control average pitch and rotor disk orientation. The higher-order terms are at our disposal to shape the angle-of-attack distribution over the rotor disk without interfering with the basic thrust and attitude commands.

In forward flight, the retreating blade is subject to lower local airspeed, and must compensate by operating at a higher angle of attack. This could be relieved by flapping, if the rotor were permitted to "blow back." However, the rotor is also a propulsion device, which must work harder as the speed increases. This dictates a forward tilt, which aggravates the disparity in angle of attack between the advancing and retreating blades. As the speed increases, sections of the retreating blade stall. One result is a significant increase in power consumption. Another is a bound on the thrust produced by the rotor, even though most sections are operating below their local capacity. In the following we show how the judicious use of second harmonic control delays blade stall, reduces the area of the rotor disk that is stalled, and makes flight possible at higher forward speed.

III. Computer Simulation

The results presented here were obtained with an adaptation of Bladehelo—a rotor blade simulation developed at the University of Alabama for implementation in our real-time helicopter simulator. The rotor model was developed by graduate student Kenneth Graham.[11,12] In the version used here, the blade is attached to a fixed shaft and subject to specified external flow.

The rotor data are those of the Arcidiacono sample helicopter[3]:

Number of blades	= 5
Blade length	= 34 ft
Blade chord	= 1.75 ft
Blade weight	= 278 lb (distributed uniformly)
Twist	= 14 deg
Airfoil section	= NACA 0012
Rotational rate	= 3.07 rev/s

The blades are attached to flapping hinges at the hub. They are not allowed to lead or lag. Full dynamic equations for the rigid blades are employed. The flapping motion of each blade is integrated in steps, while the blade itself is partitioned into elements for the purpose of computing local loads. The results shown were obtained with the blade represented as 15 elements and using integration steps covering 4 deg in azimuth. Steady-state aerodynamic data is used and uniform inflow is assumed.

Aerodynamic data used are for a Reynolds number of 6×10^6. Individual tables for Mach number varying from 0.15 to Mach 1 and beyond were used. The lift and drag coefficients for each element were obtained by interpolating in and between tables for Mach numbers bounding the actual local M.

In the manned simulator application of Bladehelo, offset hinges are necessary for controllability. The crude autopilot